Parallel self-testing of (tilted) EPR pairs via copies of (tilted) CHSH
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Caltech
arXiv 1609.03687

The parallel-repeated magic square game is rigid
Matthew Coudron
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Anand Natarajan
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arXiv 1610.03574

Overlapping qubits
Rui Chao
USC
Ben W. Reichardt
USC
Chris Sutherland
USC
Thomas Vidick
Caltech
arXiv 1701.01062
5 superconducting qubits, IBM

16 trapped ion qubits, UMD/NIST

1152 superconducting qubits, D-Wave

quantum computers are scaling up

\( n \) qubits \( \Rightarrow 2^n \) dimensions \( \Rightarrow \) exponentially hard to analyze
Quantum computers are scaling up.

$n$ qubits $\Rightarrow 2^n$ dimensions $\Rightarrow$ exponentially hard to analyze.

**How to test quantum computers?**

- **Small**
  - State/process tomography

- **Medium**
  - Error correction?
  - Small simulation?

- **Our tests!**

- **Large**
  - Factorization
Testing quantum systems

- Is it quantum?
- How many qubits?
- How much entanglement?
- How does it work?

Accept or Reject?
Goal: tests for large quantum systems that take polynomial time and/or with high probability and/or tolerate constant noise.

Testing quantum systems:
- Is it quantum?
- How many qubits?
- How much entanglement?
- How does it work?

Accept or Reject?

scalability
efficiency
completeness & soundness
robustness & rigidity
Test the dimensionality of a single quantum system
—How many qubits overlapping

Test the number of (tilted) EPR pairs between two systems
—How much entanglement

next:

• Andrea: using tilted CHSH games
• Matthew: using Magic Square games
Quantum systems are made of qubits in tensor product

$n$ qubits $\Rightarrow 2^n \text{ dim}$
Quantum systems are made of qubits \textit{in tensor product}.

\[
\begin{array}{c}
\text{n qubits } \Rightarrow 2^n \text{ dim}
\end{array}
\]

In general qubits can \textit{overlap}.

Operations on one qubit can slightly affect the others.
Quantum systems are made of qubits in tensor product

\[ n \text{ qubits } \Rightarrow 2^n \text{ dim} \]

In general qubits can overlap

operations on one qubit can slightly affect the others

\[ \| [U_1, U_2] \| \leq \epsilon \]
Quantum systems are made of qubits in tensor product

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In general qubits can overlap

\[ \| [U_1, U_2] \| \leq \epsilon \]

operations on one qubit can slightly affect the others

\[ n \epsilon\text{-overlapping qubits} \Rightarrow n^{1/\epsilon^2} \text{ dim} \]
Theorem 1:

$n$ overlapping qubits can fit in $\text{poly}(n)$ dimensions

\[ n^{1/\varepsilon^2} \] dimensions

\( \varepsilon \)-overlap

(operations on one qubit can affect any other qubit by at most \( \varepsilon \))
Theorem 1:

n overlapping qubits can fit in poly(n) dimensions

‘𝜖-overlap’

(operations on one qubit can affect any other qubit by at most 𝜖)

⇒

n^{1/𝜖^2} dimensions

Theorem 2:

Given access to n (overlapping) qubits, ∃ a test s.t.

\[ \Pr[\text{pass test}] \geq 1 - \epsilon \Rightarrow \text{dimension} \geq (1 - O(n^2 \epsilon)) \ 2^n \]
Definitions:

- A qubit in $\mathcal{H}$ is a pair of anti-commuting reflections on it.

Indeed:

\[ \{X, Z\} = 0 \Rightarrow X \simeq \sigma^x \otimes 1 \]
\[ Z \simeq \sigma^z \otimes 1 \]
Definitions:

- A qubit in $\mathcal{H}$ is a pair of anti-commuting reflections on it

\[ \mathcal{H} \simeq \mathbb{C}^2 \otimes \mathcal{H}' \]

Indeed:

\[ \{ X, Z \} = 0 \Rightarrow X \simeq \sigma^x \otimes 1 \]
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- The overlap $\varepsilon$ of 2 qubits $(X_1, Z_1), (X_2, Z_2)$ in $\mathcal{H}$ is given by

\[ \max_{P, Q \in \{ X, Z \}} \| [P_1, Q_2] \| \]
Definitions:

- **A qubit in** \( \mathcal{H} \) **is a pair of anti-commuting reflections on it**

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\( \varepsilon = 0 \Leftrightarrow \) qubits in tensor product:

\[ X_1 \simeq \sigma^x \otimes I \otimes 1 \quad X_2 \simeq I \otimes \sigma^x \otimes 1 \]
\[ Z_1 \simeq \sigma^z \otimes I \otimes 1 \quad Z_2 \simeq I \otimes \sigma^z \otimes 1 \]
Theorem 1:

$n \varepsilon$-overlapping qubits can fit in $n^{\Omega(1/\varepsilon^2)}$-dimensional Hilbert space.

Proof idea:

nearly orthogonal vectors

3n points in $\mathbb{R}^{O(\log n/\varepsilon^2)}$
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Proof idea:

- nearly orthogonal vectors
  - $\downarrow$ *group in threes*
- nearly orthogonal subspaces
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Proof idea:

- nearly orthogonal vectors
  - group in threes
- nearly orthogonal subspaces
  - Clifford algebra rep.
- nearly commuting qubits

\[ X = i E F \quad Z = i E G \]

$(n^{\Omega(1/\varepsilon^2)}$-dim ref.)
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Proof idea:

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Note: meaningful only if \( \varepsilon = \Omega(\sqrt{\log n/n}) \)
Dimension test: Given access to n qubits

1. Sequentially store n random qubits (\(|0\rangle, |1\rangle, |+\rangle, \text{or} |-\rangle\))
2. Retrieve a random index & check it’s correct

\[
\text{I} \quad \text{j} \quad \text{n}
\]
Dimension test: Given access to n qubits

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![Diagram showing overlapping circles indicating dimension test]

Theorem 2:

\[ \Pr[\text{pass test}] \geq 1 - \varepsilon \Rightarrow \text{dimension} \geq (1 - O(n^2\varepsilon)) \ 2^n \]

Note: meaningful only if \( \varepsilon = O(1/n^2) \)
Summary

- **Qubit**: anti-commuting reflection pair
- **Overlapping qubits**: nearly commuting reflections

- **Qubit packing**: 
  \[ n \text{ overlapping qubits can fit in } \text{poly}(n) \text{ dimensions} \]
- **Qubit separation**: 
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Applications and open questions:

- Test functionality
- Loosen assumptions & run experiments
- **Self-testing of EPR states**
Summary

- **Qubit**: anti-commuting reflection pair
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Thank you!