Tsirelson’s problem and linear system games

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includes joint work with Richard Cleve and Li Liu
A speculative question

Conventional wisdom: Finite time / volume / energy / etc. → can always describe nature by finite-dimensional Hilbert spaces

Could nature be “intrinsically” infinite-dimensional?

Answer: Probably not

But if it was... could we recognize that fact in an experiment? (For instance, in a Bell-type experiment?)

Tsirelson’s problem and linear system games
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Non-local games (aka Bell-type experiments)

Win/lose based on outputs $a, b$ and inputs $x, y$

Alice and Bob must cooperate to win

Winning conditions known in advance

Complication: players cannot communicate while the game is in progress
Suppose game is played many times, with inputs drawn from some public distribution $\pi$

To outside observer, Alice and Bob’s strategy is described by:

$$P(a, b| x, y) = \text{the probability of output } (a, b) \text{ on input } (x, y)$$

*Correlation matrix:* collection of numbers $\{P(a, b| x, y)\}$
What can $P(a, b|x, y)$ be?

$P(a, b|x, y) = \text{the probability of output } (a, b) \text{ on input } (x, y)$

$n$ questions, $m$ answers: $\{P(a, b|x, y)\} \subset \mathbb{R}^{m^2 n^2}$

Classically

$P(a, b|x, y) = p_a^x \cdot q_b^y$

Probability that Alice outputs $a$ on input $x$

Same for Bob

Tselelson’s problem and linear system games
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**Classically**

$$P(a, b|x, y) = \sum_i \lambda_i \cdot p_a^{x_i} \cdot q_b^{y_i}$$

- Shared randomness
- Probability that Alice outputs $a$ on input $x$
- Same for Bob

Tseilson’s problem and linear system games

William Slofstra
What can $P(a, b|x, y)$ be?

- $P(a, b|x, y) =$ the probability of output $(a, b)$ on input $(x, y)$
- $n$ questions, $m$ answers: $\{P(a, b|x, y)\} \subset \mathbb{R}^{m^2n^2}$

**Quantum**

\[
P(a, b|x, y) = \bra{\psi} M_a^x \otimes N_b^y |\psi\rangle
\]

- Alice’s measurement on input $x$
- Bob’s measurement on input $y$
- shared state on $H_1 \otimes H_2$
What can $P(a, b|x, y)$ be?

$P(a, b|x, y) =$ the probability of output $(a, b)$ on input $(x, y)$

$n$ questions, $m$ answers: $\{P(a, b|x, y)\} \subset \mathbb{R}^{m^2n^2}$

Quantum

$P(a, b|x, y) = \langle \psi | M_a^x \otimes N_b^y | \psi \rangle$

Why? axiom of quantum mechanics for composite systems
Bell inequalities

$C_c(m, n) = $ set of classical correlation matrices

$C_q(m, n) = $ set of quantum correlation matrices

Both are convex subsets of $\mathbb{R}^{m^2n^2}$. 

(all diagrams are schematic)
Bell inequalities ct’d

\[ \omega(G, P) = \omega^q(G) \]

\[ \omega(G, P) = \omega^c(G) \]

\( \omega(G, P) \) = probability of winning game \( G \) with correlation \( P \)

\( \omega^c(G) \) = maximum winning probability for \( P \in C_c(m, n) \)

\( \omega^q(G) \) = same thing but with \( C_q(m, n) \)
Bell inequalities ct’d

If $\omega^c(G) < \omega^q(G)$, then

1. $C_c \subsetneq C_q$, and
2. we can (theoretically) show this in an experiment
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(1) $C_c \subsetneq C_q$, and

(2) we can (theoretically) show this in an experiment.

Bell’s theorem + many experiments: this happens!
Finite versus infinite-dimensional

Quantum correlations:

\[ P(a, b|x, y) = \langle \psi | M^x_a \otimes N^y_b | \psi \rangle \]

where \( |\psi\rangle \in H_1 \otimes H_2 \)
Finite versus infinite-dimensional

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Correlation set \( C_q \):

\( H_1, H_2 \) must be finite-dimensional

(but, no bound on dimension)
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Correlation set \( C_{qs} \):

\( H_1, H_2 \) allowed to be infinite-dimensional

(the ‘s’ stands for ‘spatial tensor product’)

Tsirelson’s problem and linear system games
Finite versus infinite-dimensional ct’d

Can we separate $C_q$ from $C_{qs}$ with a Bell inequality?

No! This is the wrong picture.
Finite versus infinite-dimensional ct’d

Can we separate $C_q$ from $C_{qs}$ with a Bell inequality?

NO!

This is the wrong picture
How is this picture wrong?

$C_q$ and $C_{qs}$ are not known to be closed.
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Even worse: $\overline{C_{qs}} = \overline{C_q}$
How is this picture wrong?

$C_q$ and $C_{qs}$ are not known to be closed.

Even worse: $\overline{C_{qs}} = \overline{C_q}$

New correlation set $C_{qa} := \overline{C_q}$

contains limits of finite-dimensional correlations indistinguishable from $C_q$ and $C_{qs}$ in experiment
The real picture

Could look like:

We know $C_q \subseteq C_{qs} \subseteq C_{qa} \ldots$ but nothing else!
The real picture

Could look like:

\[
\begin{array}{c}
\vdots \quad \text{in } C_q \\
\text{dotted} \quad \text{in } C_{qs} \text{ but not } C_q \\
\text{dashed} \quad \text{in } C_{qa} \text{ but not } C_{qs}
\end{array}
\]

We know \( C_q \subseteq C_{qs} \subseteq C_{qa} \ldots \) but nothing else!

Fortunately, this is not the end of the story

We’ve assumed that \( \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \ldots \) maybe this is too restrictive
Commuting-operator model

Another model of composite systems

Correlation set $C_{qc}$:

$$P(a, b|x, y) = \langle \psi | M_a^x \cdot N_b^y | \psi \rangle$$

where

(1) $|\psi\rangle$ belongs to a joint Hilbert space $H$
   (possibly infinite-dimensional)

(2) Measurements commute: $M_a^x N_b^y = N_b^y M_a^x$ for all $x, y, a, b$

‘qc’ stands for ‘quantum-commuting’
What do we know about $C_{qc}$

Correlation set $C_{qc}$: 

$$P(a, b|x, y) = \langle \psi | M_a^x \cdot N_b^y | \psi \rangle$$

$C_{qc}$ is closed!

Get a hierarchy $C_q \subseteq C_{qs} \subseteq C_{qa} \subseteq C_{qc}$ of convex sets
What do we know about $C_{qc}$

Correlation set $C_{qc}$: $P(a, b|x, y) = \langle \psi | M^x_a \cdot N^y_b | \psi \rangle$

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Get a hierarchy $C_q \subseteq C_{qs} \subseteq C_{qa} \subseteq C_{qc}$ of convex sets

If $H$ is finite-dimensional, then \{ $P(a, b|x, y)$ \} $\in C_q$

Can find $H_1, H_2$ such that $H = H_1 \otimes H_2$,

$M^x_a \cong \tilde{M}^x_a \otimes I$ and $N^y_b \cong I \otimes \tilde{N}^y_b$ for all $x, y, a, b$

This argument doesn’t work if $H$ is infinite-dimensional
Tsirelson’s problem(s)

Tsirelson problems: is $C_t$, $t \in \{q, qs, qa\}$ equal to $C_{qc}$

- $C_q \subseteq C_{qs} \subseteq C_{qa} \subseteq C_{qc}$
  - strong tensor product
  - weak commuting operator

Comparing two axiom systems:
1. Strong Tsirelson: is $C_q = C_{qc}$?
2. Is $\diamondsuit_q(G) < \diamondsuit_{qc}(G)$ for any game? Equivalent to weak Tsirelson: is $C_{qa} = C_{qc}$?
Tsirelson’s problem(s)

\[ C_q \subseteq C_{qs} \subseteq C_{qa} \subseteq C_{qc} \]

- **strong**
- **tensor product**
- **weak**
- **commuting operator**

Tsirelson problems: is \( C_t, t \in \{q, qs, qa\} \) equal to \( C_{qc} \)

These are fundamental questions

1. Comparing two axiom systems:
   - Strong Tsirelson: is \( C_q = C_{qc} \)?
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These are fundamental questions

1. Comparing two axiom systems:
   - Strong Tsirelson: is $C_q = C_{qc}$?
2. Is $\omega^q(G) < \omega^{qc}(G)$ for any game?
   - Equivalent to weak Tsirelson: is $C_{qa} = C_{qc}$?
What do we know?

Theorem (Ozawa, JNPPSW, Fr)

\[ C_{qa} = C_{qc} \text{ if and only if Connes’ embedding problem is true} \]
What do we know?

\[ C_q \subseteq C_{qs} \subseteq C_{qa} \subseteq C_{qc} \]

Theorem (Ozawa, JNPPSW, Fr)

\[ C_{qa} = C_{qc} \text{ if and only if Connes’ embedding problem is true} \]

Theorem (S)

\[ C_{qs} \neq C_{qc} \]
Other fundamental questions

1 Resource question:

A non-local game $G$ is a computational task
Bell’s theorem: can do better with entanglement
Can $G$ be played optimally with finite Hilbert space dimension?

Yes $\iff C_q = C_q^a$ (in other words, is $C_q$ closed?)

Variants of games: finite dimensions do not suffice
[LTW13],[MV14],[RV15]
Other fundamental questions

1. Resource question:
   A non-local game $G$ is a computational task
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   Can $G$ be played optimally with finite Hilbert space dimension?
   Yes $\iff C_q = C_{qa}$ (in other words, is $C_q$ closed?)
   Variants of games: finite dimensions do not suffice
   [LTW13],[MV14],[RV15]

2. Can we compute $\omega^q(G)$ or $\omega^{qc}(G)$?
   (what is the power of $MIP^*$?)
What do we know?

Question: can we compute $\omega^q(G)$ or $\omega^{qc}(G)$?

Brute force search through strategies on $HA = HB = CN$, converges to $\omega^q$ (from below)

Navascu´es, Pironio, Ac ´ın: Given a non-local game, there is a hierarchy of SDPs which converge in value to $\omega^{qc}$ (from above)

In both cases, no way to tell how close we are to the correct answer

Theorem (S)

It is undecidable to tell if $\omega^{qc} < 1$

General cases of other questions completely open!
What do we know?

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Brute force search through strategies on $\mathcal{H}_A = \mathcal{H}_B = \mathbb{C}^n$, converges to $\omega^q$ (from below)

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NPA hierarchy: there is no computable function

$$L : \text{Games} \rightarrow \mathbb{N}$$

such that $\omega^{qc}(G) = L(G)$th level of NPA hierarchy.
Undecidability

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NPA hierarchy: there is no computable function

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such that $\omega^{qc}(G) = L(G)$th level of NPA hierarchy

We still don’t know: can we compute $\omega^{qc}(G)$ to within some given error?

(Ji ’16: this problem is $MIP^*$-complete)

If weak Tsirelson is true, then $\omega^{qc}$ is computable in this stronger sense
Undecidability comes from exact error?

Comparison point: Can decide if optimal value of finite SDP is $< 1$ (very inefficient algorithm)
Undecidability comes from exact error?

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More generally: first-order logic for field of real numbers is decidable

Contrast: first-order logic for integers and rationals is undecidable

Consequence of undecidability of $\phi_C$ due to Tobias Fritz:

Quantum logic (first order theory for projections on Hilbert spaces) is undecidable
Undecidability comes from exact error?

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Consequence of undecidability of $\omega^{qc} < 1$ due to Tobias Fritz:

quantum logic (first order theory for projections on Hilbert spaces) is undecidable
Quantum logic is undecidable

**Theorem (Tobias Fritz)**

The following problem is undecidable: Given \( n \geq 1 \) and a collection of subsets \( C \) of \( \{1, \ldots, n\} \), determine if there are self-adjoint projections \( P_1, \ldots, P_n \) such that

\[
\sum_{i \in S} P_i = I, \quad P_i P_j = P_j P_i = 0 \text{ if } i \neq j \in S
\]

for all \( S \in C \).

Proof: follows from undecidability of \( \omega^{qc} < 1 \)

Builds on Acín-Fritz-Leverrier-Sainz '15.
Two theorems

<table>
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Theorems look very different...
Two theorems

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Theorem (S)

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Theorems look very different...

But: proof follows from a single theorem in group theory

Connection with group theory comes from linear system games
Linear system games

Start with \( m \times n \) linear system \( Ax = b \) over \( \mathbb{Z}_2 \)

Inputs:
- Alice receives 1 equation
- Bob receives 1 variable

Outputs:
- Alice outputs an assignment \( a_k \) for all variables \( x_k \) with \( A_{ik} \neq 0 \)
- Bob outputs an assignment \( b_j \) for \( x_j \)

They win if:
- \( A_{ij} = 0 \) (assignment irrelevant) or
- \( A_{ij} \neq 0 \) and \( a_j = b_j \) (assignment consistent)

Such games go back to Mermin-Peres magic square, more recently studied by Cleve-Mittal, Ji, Arkhipov.
Linear system games

Start with $m \times n$ linear system $Ax = b$ over $\mathbb{Z}_2$

Inputs:
- Alice receives $1 \leq i \leq m$ (an equation)
- Bob receives $1 \leq j \leq n$ (a variable)

Outputs:
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Quantum solutions of $Ax = b$

Observables $X_j$ such that

1. $X_j^2 = I$ for all $j$
2. $\prod_{j=1}^{n} X_j^{A_{ij}} = (-I)^{b_i}$ for all $i$
3. If $A_{ij}, A_{ik} \neq 0$, then $X_j X_k = X_k X_j$

(We’ve written linear equations multiplicatively)
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(We’ve written linear equations multiplicatively)

Theorem (Cleve-Mittal, Cleve-Liu-S)

Let $G$ be the game for linear system $Ax = b$. Then:

- $G$ has a perfect strategy in $C_{qs}$ if and only if $Ax = b$ has a finite-dimensional quantum solution
- $G$ has a perfect strategy in $C_{qc}$ if and only if $Ax = b$ has a quantum solution
The solution group $\Gamma$ of $Ax = b$ is the group generated by $X_1, \ldots, X_n, J$ such that

1. $X_j^2 = [X_j, J] = J^2 = e$ for all $j$
2. $\prod_{j=1}^n X_j^{A_{ij}} = J^{b_i}$ for all $i$
3. If $A_{ij}, A_{ik} \neq 0$, then $[X_j, X_k] = e$

where $[a, b] = aba^{-1}b^{-1}$, $e =$ group identity

**Theorem (Cleve-Mittal,Cleve-Liu-S)**

Let $G$ be the game for linear system $Ax = b$. Then:

- $G$ has a perfect strategy in $C_{qs}$ if and only if $\Gamma$ has a finite-dimensional representation with $J \neq I$
- $G$ has a perfect strategy in $C_{qc}$ if and only if $J \neq e$ in $\Gamma$
Groups and local compatibility

Suppose we can write down any group relations we want…

But: generators in the relation will be forced to commute!
Groups and local compatibility

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Call this condition \textit{local compatibility}

Local compatibility is (a priori) a very strong constraint
Groups and local compatibility

Suppose we can write down any group relations we want...

But: generators in the relation will be forced to commute!

Call this condition *local compatibility*

Local compatibility is (a priori) a very strong constraint

For instance, $S_3$ is generated by $a, b$ subject to the relations

$$a^2 = b^2 = e, \ (ab)^3 = e$$

If $ab = ba$, then $(ab)^3 = a^3 b^3 = ab$

So relations imply $a = b$, and $S_3$ becomes $\mathbb{Z}_2$
Group embedding theorem

Solution groups satisfy local compatibility

Nonetheless:

Solution groups are as complicated as general groups

**Theorem (S)**

Let $G$ be any finitely-presented group, and suppose we are given $J_0$ in the center of $G$ such that $J_0^2 = e$.

Then there is an injective homomorphism $\phi : G \hookrightarrow \Gamma$, where $\Gamma$ is the solution group of a linear system $Ax = b$, with $\phi(J_0) = J$. 

Tsirelson’s problem and linear system games
How do we prove the embedding theorem?

Linear system $Ax = b$ over $\mathbb{Z}_2$ equivalent to labelled hypergraph:

Edges are variables

Vertices are equations

$v$ is adjacent to $e$ if and only if $A_{ve} \neq 0$

$v$ is labelled by $b_i \in \mathbb{Z}_2$
How do we prove the embedding theorem?

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$v$ is adjacent to $e$ if and only if $A_{ve} \neq 0$

$v$ is labelled by $b \in \mathbb{Z}_2$

Given finitely-presented group $G$, we get $\Gamma$ from a linear system

But what linear system?

Can answer this pictorially by writing down a hypergraph?
The hypergraph by example

\[ \langle x, y, z, u, v : xyxz = xuvu = e = x^2 = y^2 = \cdots = v^2 \rangle \]
Further directions

1. Further refinements to address $C_q$ vs $C_{qa}$

2. Is $\omega^q(G) < 1$ decidable?
Further directions

1. Further refinements to address $C_q$ vs $C_{qa}$

2. Is $\omega^q(G) < 1$ decidable?

3. Embedding theorem: for any f.p. group $G$, get a non-local game such that Alice and Bob are forced to use $G$ to play perfectly

(Caveat: but might need to use infinite-dimensional commuting-operator strategy to achieve this)

Applications to self-testing / device independent protocols?
Thank-you!
Extra slide: Higman’s group

\[ G = \langle a, b, c, d : aba^{-1} = b^2, bcb^{-1} = c^2, cdc^{-1} = d^2, dad^{-1} = a^2 \rangle \]

Only finite-dimensional representation is the trivial representation.

On the other hand, \( a, b, c, d \) are all non-trivial in \( G \).